## All about "CHIPS" (Chipstead High Point Scoring)

## 1. Introduction

This paper outlines the history of the development of the CHIPS race scoring system, which was developed at Chipstead Sailing Club and is now finding favour at a number of other clubs. The development is based upon a combination of mathematics and practical experience gained from analysing the outcome of a large number of multi-race sailing events.

While the paper describes how the apparently complicated mathematical formulae evolved to what is now considered to be the optimum version of CHIPS, the rationale for this evolution is based primarily on the subjective views of competing sailors when judging the fairness of scores in over 850 races sailed during 2005 and 2006.

## 2. High Point Scoring Background

The ISAF Low Point Scoring method, used by the majority of sailing clubs, works best for regattas with a limited number of races, when most races are required to count and when the number of competitors remains broadly similar. There is a strong body of opinion in the international sailing community that such a scoring method fails to measure racing performance fairly when used to score a long series in which the number of competitors varies substantially from race to race. In such cases, many yacht/sailing clubs have moved to "High Point Scoring" to yield fairer results.

Common sense supports the idea that anyone that beats more boats in a race should get a greater reward. The Low Point system does the reverse of this by always allocating the winning boat 1 point and, instead, penalising those at the back of the fleet by giving them a larger points score when more boats beat them. It may appear that the two approaches amount to the same thing but mathematically this is not the case when the results are combined into a series. The scores that determine an overall series winner are clearly those gained in the top part of the fleet. It is clearly more important to give particular attention to the achievement of fair scores for those in contention for prizes. In High Point Scoring, emphasis is placed first on setting an appropriate score for the winning boat by measuring how well she sailed compared to the other boats - this of course depends on how many others participate in the race. The score for the last boat is then set at a low figure, with the scores for all the other boats being slotted in uniformly between the first and last scores. The reference here to a "low figure" for the last boat is to allow for setting an even lower score for retirements (non-finishers), disqualifications and non-starters.

In reality the choice of a scoring system is entirely arbitrary and inevitably based upon a subjective perception of what is considered to be the fairest scoring method for any particular event, depending on the number of races in the series, a judgement as to the number of discards that should be allowed and whether all (or nearly all) competitors are expected to sail in all races. The fairness of the scoring system only becomes an issue when the event comprises a series of races; for a single-race event the choice of scoring system becomes totally unimportant since, then, only the positions matter. In major sailing events there is a tendency to limit the number of discards that are permitted, creating an incentive for boats to sail all races, in which case a High Point Scoring method offers few advantages.

There are a number of variants of High Point Scoring but ideally the system should aim to:

- Give an appropriate score for winning based upon the degree of competitiveness in the race.
- Reflect the fact that it is harder to do well in a race with more competing boats.
- Recognise that the tail-enders do not normally sail less well in the presence of more boats and should not therefore be unduly penalised by their score being determined principally by the number of participating boats.
- Minimise the likelihood of a series tie. At many clubs, such as Chipstead SC, a large number of discards are permitted in a long series, which means that there is an increased likelihood of ties near the top of the fleet. Likewise in a short series - such as a 3-race event in which 2 are to count - it is common for a tie to occur when conventional Low Point Scoring is used; in accordance with ISAF rules, the winner is then likely to be determined by which boat did better in the final race, often giving an unsatisfactory outcome based upon a race in which the competition is less severe (e.g. with fewer possibly boats, as tends to occur when a number of the competitors decide not to sail because they are no longer in contention.).
- Create an incentive to race and give fair scores at all positions within the fleet. The achievement of a fair competition for those in the middle of the fleet is considered to be as important to competitors as it is for those striving to win.


## 3. Rinderle B Scoring System

A commonly used simple method of High Point Scoring allocates a points score for each boat equal to the number of boats beaten plus one. However schemes of that type go too far in the wrong direction by giving an excessively high score for the winner: a more sensitive method is required to achieve a fair outcome.

A method that has gained substantial support internationally is the "Rinderle B" system, which was invented in the early 1980's by Jim Rinderle of Marblehead, USA, to make a more equitable way of scoring a series of races that varied considerably in size. Rinderle originally developed a number of candidate scoring tables, dubbed Rinderle A, B and C, etc. The consensus of a group of race organizers was that Rinderle B seemed to balance the above somewhat competing aims. This scoring system was developed to weight performance as a function of two separate criteria. The first is the finishing position. The second is the number of boats participating in a given race. The Rinderle B scores are available only in the form of a table, which can be found at (http://www.gmora.org/pdf/rinderle b table.PDF).

While it treats winning boats reasonably, Rinderle B is somewhat aggressive in its behaviour in that it penalises boats further down the fleet excessively when there are only a few competitors. This is because it allocates the same poor score to the last placed boat irrespective of the number of competitors.

In addition the scheme has a mathematical anomaly that becomes apparent if one inspects the behaviour of the first and last place scores as one reduces the number of boats. Those scores should converge to the same figure for a single-boat race, whereas the Rinderle B scores fail to converge because the last place score remains as 10.5.

To overcome the undesirability of using a look-up table I devised a formula that behaves similarly to Rinderle B but with the scores having been re-scaled to give a maximum of 100 points for winning a race with many competitors. Note however that, while this "modified" version of Rinderle B retains the same shape as the scores originally published in tabular form, the values are slightly different. The Modified Rinderle B formula is:

$$
S_{p, n}=89.5 \times\left(1-e^{-(0.8+0.23 n)}\right) \frac{(n-p)}{(n-1)}+10.5
$$

$\boldsymbol{S}_{\boldsymbol{p , n} \boldsymbol{n}} \quad$ is the points scored for position $\boldsymbol{p}$ in a race of $\boldsymbol{n}$ boats
Rinderle B scores 5 points for a retirement.

## 4. Original Derivation of CHIPS

CHIPS was devised in order to address the undesirable features, while retaining the principles of Rinderle B and creating a scoring system that gives a fairer outcome than conventional Low Pont Scoring for club racing. While Rinderle gives reasonable scores for winning, the need was identified to treat all the lower positions in a less aggressive manner, as becomes evident when only a few boats sail.

A study of the mathematics indicated that enhancements could be made to the modified Rinderle B formula (above), leading to the following generic formula:

$$
S_{p, n}=(100-d)\left\{\left[\frac{(n-p)}{(n-1)}\right]^{k}\left[1-(1+f) e^{-(b+c n)}\right]+f e^{-(b+c n)}\right\}+d
$$

While this "generic" formula appears complicated, it is nonetheless convenient to be aware of the original mathematical derivation of CHIPS in order to be able to understand how to adjust the various components to change the shape of the score curves. The curve shape is determined by the constants $\boldsymbol{b}, \boldsymbol{c}, \boldsymbol{d}, \boldsymbol{f}$ and $\boldsymbol{k}$, which contribute as follows:
$\boldsymbol{b}$ and $\boldsymbol{c}$ are constants that, together, determine the amount of curvature through their effect on the score for a first place $(\boldsymbol{p}=\mathbf{1})$ and the score increment between positions.
$\boldsymbol{k}$ is a constant that determines the amount of non-linearity as a function of position. When $\boldsymbol{k}=\mathbf{1}$, the score increment remains constant between all positions (for any particular number of boats, $n$ ), while if $\boldsymbol{k}$ exceeds 1 then the score difference diminishes as one progresses down the fleet. The purpose of this is to create an increased score advantage to those at the top of the fleet with the greatest advantage being allocated to the winner. (While retained in the generic formula this non-linear feature has since been discovered to have no great merit since, for sensible values when $\boldsymbol{k}=\mathbf{1}$, there is always a significant advantage in winning as opposed to coming $2^{\text {nd }}-i . e$. the loss against a maximum of 100 is less for coming first than is the increment between the scores. For this reason and for simplicity, the constant $\boldsymbol{k}$ has been set to unity for all versions of CHIPS used to date.)
$\boldsymbol{d}$ is the score allocated to last place in a race with a large number of boats (in theory when $\boldsymbol{n}=\infty$ ).
$f$ is a constant that determines the way in which the score for last place changes with the number of boats in the race. Just as the score for winning reduces as the number of boats, $\boldsymbol{n}$, decreases, the last place score correspondingly increases following a similarly shaped curve. $\boldsymbol{f}$ is the factor that determines the relationship between the first and last place curves. If, for example, $\boldsymbol{f}$ is set to equal to 1.5 then the last place score increase above $\boldsymbol{d}$ by 1.5 times the amount that the winning score decreases from 100.

It is the introduction of the important factor $\boldsymbol{f}$ that distinguishes CHIPS from Rinderle, the purpose being to allocate fairer scores to all competitors not just the winner. If $\boldsymbol{f}$ is set to zero, the formula reverts to the "modified Rinderle B" equation. It should be noted that, except for certain conditions, the $\boldsymbol{f}$ factor tends to a pair of different values for the points score as the number of boats approaches 1. (The latter represents a hypothetical oneboat race and needs care in avoiding a division by zero in the formula).

At that stage of the development of CHIPS (i.e. 2005), the Rinderle B approach for scoring Retirements was retained through the adoption of the following formula:

$$
S_{R T D, n}=\frac{5}{10.5} S_{n, n}
$$

That is, the score for a Retirement remains just less than half the score for a last place, using the same proportion as for Rinderle B. (This principle was however later changed in the light of experience gained during the 2005 season.)
This version of CHIPS (referred to as "CHIPS 1") was applied on a trial basis for all club racing at Chipstead SC for the 2005 season, for 27 series and 491 races, with the following values for the constants:

$$
\begin{aligned}
& \boldsymbol{b}=0.81 \\
& \boldsymbol{c}=0.23 \\
& \boldsymbol{d}=10.5 \\
& \boldsymbol{f}=1.478 \\
& \boldsymbol{k}=1.0
\end{aligned}
$$

Curves for the above CHIPS 1 values are shown at the end of this paper as a comparison with "modified" Rinderle B and subsequent versions of CHIPS.

Note that, for all versions of CHIPS, the scores should rounded to the nearest 0.1 to avoid rounding errors (i.e. not simply displayed to 0.1).

## 5. Evolution to CHIPS 2

Conclusions from the 2005 trial of CHIPS 1 were:

- The chosen values for the constants yielded an overly aggressive reduction in the score as the number of boats decreases. This became apparent in a long series when the system discarded apparently good results while retaining apparently worse positions gained when there were only a few additional boats sailing. While this is a correct implementation and inherent in the design of systems like Rinderle $B$ and CHIPS, some competitors complained that this occurred too frequently..
- It was felt that retiring boats were penalised excessively in receiving less than half the score they would have gained had they finished. The score reduction for failing to finish was substantially greater than the increment between places and this appeared to be too severe a penalty.
- Some people felt that there was no merit in sailing in heavy weather since only the better sailors would be likely to sail and they would inevitably score relatively badly even for a win.
In order to address the above for the 2006 season four changes were introduced:
(a) The curves were "flattened" to reduce the dependence on the number of boats. This was achieved by adjusting the constants $\boldsymbol{b}$ and $\boldsymbol{c}$.
(b) The points for a Retirement were changed to be equivalent to the score for coming Last +1 . That is, $S_{R T D, n}$ became $S_{n+1, n}$, calculated by setting $\boldsymbol{p}=\boldsymbol{n + 1}$.
(c) In the event of heavy weather the scores were calculated by artificially increasing the number of starters ( $\boldsymbol{n}$ ) in the formula by 2. This is equivalent to including two "phantom boats" in the race. A definition of "heavy weather" was added to the Chipstead SC rules, involving wind strength measurements to ensure this provision was applied objectively. (While this "heavy weather" provision currently remains for sailing at Chipstead SC, in practice there appears little merit in its inclusion and I would recommend to other users of CHIPS to therefore ignore this addition, which is not a requirement of the scoring system itself).
(d) The constant $f$ was eliminated by solving the equations to ensure the requirement for the $\boldsymbol{p}=1$ and the score for a last place were made equal in the hypothetical single boat race when $\boldsymbol{n}=1$. This eliminated the residual anomaly inherited from Rinderle B.

$$
S_{p, n}=(100-d)\left[\frac{(n-p)}{(n-1)}\left(1-e^{-c(n-1)}\right)+e^{-c n}\left(e^{c}-e^{-b}\right)\right]+d
$$

Note that, although the above formula appears to differ from the original CHIPS 1 version, this is not the case: it is precisely the same formula but with the constant $\boldsymbol{f}$ having been eliminated by solving the equations while retaining the option to still adjust the constants $\boldsymbol{b}$ and $\boldsymbol{c}$ to allow the shape of the curves to be adjusted further if desired.

The values for the constants that were considered to give the optimum shape for the curves were:

$$
\begin{aligned}
& \boldsymbol{b}=1.713 \\
& \boldsymbol{c}=0.163 \\
& \boldsymbol{d}=10
\end{aligned}
$$

The equation can then be evaluated into the following simpler form:

$$
S_{p, n}=90 \times\left[\frac{(n-p)}{(n-1)}\left(1-1.177 e^{-0.163 n}\right)+0.99671 e^{-0.163 n}\right]+10
$$

This version, identified with the name "CHIPS 2", was used for all club racing at Chipstead SC during 2006 and up until $25^{\text {th }}$ March 2007.

## 6. CHIPS 3

The 2006 results indicated that no changes were required to CHIPS 2 for the number of boats that normally sail at Chipstead (less than 30 per race). Although for the 2007 season Chipstead is retaining the "heavy weather" feature, involving 2 phantom boats, this was used only twice during 2006 and this had no impact on the final series results.

However, Malcolm Clark (Banbury SC) has undertaken a detailed mathematical analysis of CHIPS and identified an anomaly in the scores that are allocated for a Retirement when the number of boats exceeds 31 (http://www.styvechale.net/chip.pdf). The Retirement anomaly is readily overcome by amending the formula slightly without significantly altering the scores for race winners or when there is a low number of boats. This revision has led to the following CHIPS 3 formula.

$$
S_{p, n}=95 \times\left[\frac{(n+1-p)}{n}\left(1-0.986682 e^{-0.1622 n}\right)+0.81475 e^{-0.1622 n}\right]+5
$$

RTD is scored as $\boldsymbol{p}=\boldsymbol{n}+\mathbf{1}$. Note that this relates to DNF (Did Not Finish) and RAF (Retired After Finishing).
DNC (Did not come to the starting area), DNS (Did Not Start), and Disqualifications (including for example OCS) all score zero points.
Key features of the CHIPS 3 formula are that $1^{\text {st }}, 2^{\text {nd }}$ and $3^{\text {rd }}$ places in a 3 boat race are unchanged from CHIPS 2 and scored as $90,75.5$ and 65 points, respectively, also that a Retirement asymptotically reduces towards 5 points as the number of boats increases.

A by-product of the revised formula is that the "divide by zero" problem no longer arises when $\boldsymbol{n}=1$.

Rewriting this as a custom formula as needed for direct use in Sailwave (http://www.sailwave.com/), and designating $s$ as the number of starters (replacing $n$ ) it becomes:

5+95*(((s+1-p)/s)*(1-0.986682*2.7183^(-0.1622*s))+0.81475*2.7183^(-0.1622*s))
A Retirement (DNF or RAF) is scored as $\boldsymbol{p}=\boldsymbol{s} \boldsymbol{+ 1}$.
As can be seen from the graphs at the end of this paper the scores for CHIPS 3 are almost the same as for CHIPS 2 for races with less than 20 boats.

CHIPS 3 is the version recommended for use by all clubs.

## 7. Key Features of CHIPS

- Scores are related to Performance against the whole fleet. The Performance of helms in a large fleet is representative of the skills within the overall club/fleet and can be presented meaningfully as a percentage score.
- For finishing in any particular position in a race:
- Score more points for beating more boats
- Score fewer points if beaten by more boats.
- The race organizers set the maximum number of races that are counted. (When that number of races is exceeded the lowest scores are discarded.)
- The highest overall points score wins the race series.
- A helm that does not sail scores zero points
- A Disqualification scores zero points.
- Boats that do not finish are allocated a RTD score equivalent to finishing in "last plus one" place. (For 46 or more boats a Retirement scores 5 points.)
- A race winner scores 100 points when there are 36 or more boats.
- Ties are rarely encountered. (In Low Point Scoring ties occur frequently and are often not resolvable in a fair manner, usually requiring the adoption of some arbitrary and unsatisfactorily tie-resolution method, such as determining the winner from relative positions in the last race).
- Treats race-to race variations in fleet size in a much fairer manner than conventional Low Point Scoring.
- Since scores accumulate immediately, starting from the first race, competitors can see their positions in the fleet at all times throughout the series.
- Race organizers have the option of requiring helms to "qualify" by sailing a certain number of races in the series, or can simply calculate series positions based upon the number of races actually sailed without any "qualification" requirement. In the latter case a better sailor who has sailed fewer races may score higher than a "lesser" sailor.
- The only significant down-side is that this method of scoring is more difficult to understand than Low Point Scoring and it is more difficult to work out what a competitor needs to achieve in order to overtake another boat in the series or avoid being overtaken. To simplify this, an Excel spreadsheet version of CHIPS 3 is available for downloading from the Chipstead SC website; this calculates the points score for any combination of position and number of boats, together with a useful facility that enables helms to determine what results are needed in order to beat their nearest competitors (http://www.chipsteadsc.org.uk/chips/chips.htm).


## 8. Comparative Graphs

The following graphs show how the scoring schemes evolved from Rinderle B, through CHIPS 1, CHIPS 2 and ultimately to CHIPS 3 as the optimum version.

## High Point Scores for Each Finishing Position (Comparison of CHIPS 1 with "Modified" Rinderle B)




Note the similarity of the two schemes, except that CHIPS 1 adjusts the last-place scores as the number of boats changes.

High Point Scores for Each Finishing Position (CHIPS 3 comparison with CHIPS 2)



Note that the two schemes are almost identical. There are some small differences at the rear of the fleet when there are more than 20 boats.

## CHIPS 3 - TABLE OF POINTS

|  |  | Number of Starters (N) |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 |
|  | 1 | 90 | 91.5 | 92.7 | 93.8 | 94.8 | 95.5 | 96.2 | 96.8 | 97.3 | 97.7 | 98 | 98.3 | 98.6 | 98.8 | 99 | 99.1 | 99.3 | 99.4 | 99.5 | 99.5 | 99.6 | 99.7 | 99.7 |
|  | 2 | 77.5 | 80 | 82.1 | 83.9 | 85.5 | 86.9 | 88.1 | 89.1 | 90.1 | 90.9 | 91.6 | 92.2 | 92.8 | 93.3 | 93.7 | 94.1 | 94.5 | 94.8 | 95.1 | 95.3 | 95.6 | 95.8 | 96 |
|  | 3 | 65 | 68.5 | 71.4 | 74 | 76.2 | 78.2 | 79.9 | 81.5 | 82.8 | 84.1 | 85.2 | 86.1 | 87 | 87.8 | 88.5 | 89.1 | 89.7 | 90.2 | 90.7 | 91.1 | 91.5 | 91.9 | 92.2 |
|  | 4 |  | 57 | 60.7 | 64 | 66.9 | 69.5 | 71.8 | 73.8 | 75.6 | 77.3 | 78.7 | 80 | 81.2 | 82.3 | 83.2 | 84.1 | 84.9 | 85.7 | 86.3 | 86.9 | 87.5 | 88 | 88.5 |
|  | 5 |  |  | 50.1 | 54.1 | 57.7 | 60.8 | 63.7 | 66.2 | 68.4 | 70.5 | 72.3 | 73.9 | 75.4 | 76.8 | 78 | 79.1 | 80.2 | 81.1 | 82 | 82.7 | 83.5 | 84.2 | 84.8 |
|  | 6 |  |  |  | 44.2 | 48.4 | 52.2 | 55.5 | 58.5 | 61.2 | 63.7 | 65.9 | 67.8 | 69.6 | 71.3 | 72.8 | 74.1 | 75.4 | 76.5 | 77.6 | 78.5 | 79.4 | 80.3 | 81 |
|  | 7 |  |  |  |  | 39.1 | 43.5 | 47.4 | 50.9 | 54 | 56.9 | 59.4 | 61.7 | 63.9 | 65.8 | 67.5 | 69.1 | 70.6 | 72 | 73.2 | 74.4 | 75.4 | 76.4 | 77.3 |
|  | 8 |  |  |  |  |  | 34.8 | 39.3 | 43.2 | 46.8 | 50.1 | 53 | 55.7 | 58.1 | 60.3 | 62.3 | 64.1 | 65.8 | 67.4 | 68.8 | 70.2 | 71.4 | 72.5 | 73.6 |
|  | 9 |  |  |  |  |  |  | 31.1 | 35.6 | 39.6 | 43.3 | 46.6 | 49.6 | 52.3 | 54.8 | 57.1 | 59.1 | 61.1 | 62.8 | 64.5 | 66 | 67.3 | 68.6 | 69.8 |
|  | 10 |  |  |  |  |  |  |  | 27.9 | 32.4 | 36.5 | 40.1 | 43.5 | 46.5 | 49.3 | 51.8 | 54.1 | 56.3 | 58.3 | 60.1 | 61.8 | 63.3 | 64.8 | 66.1 |
| $\overparen{0}$ | 11 |  |  |  |  |  |  |  |  | 25.2 | 29.7 | 33.7 | 37.4 | 40.7 | 43.8 | 46.6 | 49.2 | 51.5 | 53.7 | 55.7 | 57.6 | 59.3 | 60.9 | 62.4 |
| $E$ | 12 |  |  |  |  |  |  |  |  |  | 22.9 | 27.3 | 31.3 | 34.9 | 38.3 | 41.3 | 44.2 | 46.7 | 49.1 | 51.3 | 53.4 | 55.2 | 57 | 58.6 |
| 은 | 13 |  |  |  |  |  |  |  |  |  |  | 20.8 | 25.2 | 29.1 | 32.8 | 36.1 | 39.2 | 42 | 44.6 | 46.9 | 49.2 | 51.2 | 53.1 | 54.9 |
| $\pm$ | 14 |  |  |  |  |  |  |  |  |  |  |  | 19.1 | 23.4 | 27.3 | 30.9 | 34.2 | 37.2 | 40 | 42.6 | 45 | 47.2 | 49.2 | 51.2 |
| $0$ | 15 |  |  |  |  |  |  |  |  |  |  |  |  | 17.6 | 21.8 | 25.6 | 29.2 | 32.4 | 35.4 | 38.2 | 40.8 | 43.2 | 45.4 | 47.4 |
|  | 16 |  |  |  |  |  |  |  |  |  |  |  |  |  | 16.3 | 20.4 | 24.2 | 27.6 | 30.9 | 33.8 | 36.6 | 39.1 | 41.5 | 43.7 |
|  | 17 |  |  |  |  |  |  |  |  |  |  |  |  |  |  | 15.1 | 19.2 | 22.9 | 26.3 | 29.4 | 32.4 | 35.1 | 37.6 | 40 |
|  | 18 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | 14.2 | 18.1 | 21.7 | 25.1 | 28.2 | 31.1 | 33.7 | 36.2 |
|  | 19 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | 13.3 | 17.2 | 20.7 | 24 | 27 | 29.9 | 32.5 |
|  | 20 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | 12.6 | 16.3 | 19.8 | 23 | 26 | 28.8 |
|  | 21 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | 11.9 | 15.6 | 19 | 22.1 | 25 |
|  | 22 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | 11.4 | 14.9 | 18.2 | 21.3 |
|  | 23 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | 10.9 | 14.3 | 17.5 |
|  | 24 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | 10.5 | 13.8 |
|  | 25 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | 10.1 |
|  | RTD | 52.6 | 45.5 | 39.4 | 34.2 | 29.9 | 26.1 | 23 | 20.3 | 18 | 16.1 | 14.4 | 13 | 11.8 | 10.8 | 9.9 | 9.2 | 8.6 | 8 | 7.6 | 7.2 | 6.9 | 6.6 | 6.3 |

